SIFATLI TA'LIM – YORQIN KELAJAK
ISSN: 3060-4761
TerDPI ILMIY XABARNOMASI / SCIENTIFIC BULLETIN OF THE TerSPI / НАУЧНЫЙ ВЕСТНИК ТерГПИ

UDK:621.397.64.048
ORCID: 0009-0009-9247-3510

MAMATOVA ZILOLAKHON KHABIBULLOKHONOVNA,
Associate Professor, Fergana State University
Doctor of Philosophy (PhD) in Pedagogical Sciences
E-mail: mamatovazilolakhon@gmail.com
MUHAMMADVALIYEVA MOHICHEHRA ZUHRIDDINOVNA,
Student, Fergana State University
E-mail: mohichehrazuhriddinovna006@gmail.com

MATHEMATICAL OPTIMIZATION MODEL FOR MINIMIZING REPLACEMENT AND MAINTENANCE COSTS OF DENTAL COMPRESSORS IN DENTAL CLINICS (9-YEAR ANALYSIS)

ABSTRACT: In this article, the problem of minimizing the costs associated with replacing and maintaining dental compressors used in dental clinics is mathematically modeled within a 9-year planning horizon. The cost structure of a dental compressor over its operational lifetime — including the initial purchase price, the annually increasing maintenance and repair expenses, and the residual value at the end of its service period — is comprehensively taken into account. To determine the optimal operating strategy that ensures minimal total cost, a recursive dynamic programming algorithm was developed and applied to the 9-year period. Practical calculations carried out in Microsoft Excel made it possible to identify in which years it is economically reasonable to repair the dental compressor and in which years it is preferable to replace it. The results of the study hold significant practical value for scientifically planning the operation of technological equipment in dental clinics, optimizing depreciation policy, and reducing the overall financial burden.

KEYWORDS: *dental compressor, equipment replacement, maintenance costs, dynamic programming, directed acyclic graph (DAG), operating costs, mathematical optimization, dental clinic, Excel calculations, 9-year planning.*

INTRODUCTION. In modern dental clinics, the quality of services and the continuity of technological processes primarily depend on the reliable operation of dental compressors. Dental compressors serve as the main technological equipment that provides a stable air supply to all dental devices in the clinic. Over time, the technical condition of the compressor deteriorates, the number of malfunctions increases, and maintenance costs rise. At the same time, the price of new compressors varies under the influence of technological modernization, market conditions, and inflation. Under such circumstances, clinic management faces a crucial question: when is it more appropriate to continue repairing a dental compressor, and when should it be replaced with a new unit?

This issue is of strategic importance for dental clinics, as incorrect decisions can negatively affect operational stability, service quality, and financial performance. Therefore, analyzing decisions related to the replacement and maintenance of dental compressors using mathematical optimization models is a pressing task.

The main costs incurred during the life cycle of a dental compressor consist of three components: the purchase price of a new compressor, maintenance and repair costs during operation, and the residual value at the end of its service life. By taking these components into account comprehensively, it is possible to determine the optimal replacement–maintenance strategy, i.e., to construct a mathematical model that minimizes total operating costs over a 9-year period.

Mathematical Model

The objective of the dental compressor replacement and maintenance problem is to minimize the total costs over the planning horizon. For this purpose, the following notations are introduced:

N — planning horizon (number of years);

P — purchase cost of a new unit in year i;

r — repair cost in year j during the use of the equipment;

v — resale value (residual value) of the equipment after year j.

If the equipment is purchased in year i and used until year j , the cost for this period is determined as follows:

$$c_{i-1\,j} = P_i + r_{j-i+1} - v_{j-i+1}$$
$$i, j = 1, 2, \ldots, N; i \le j \qquad (1)$$

ISSN: 3060-4761
TerDPI ILMIY XABARNOMASI / SCIENTIFIC BULLETIN OF THE TerSPI / НАУЧНЫЙ ВЕСТНИК ТерГПИ

Here:

P_i — purchase price of a new unit;

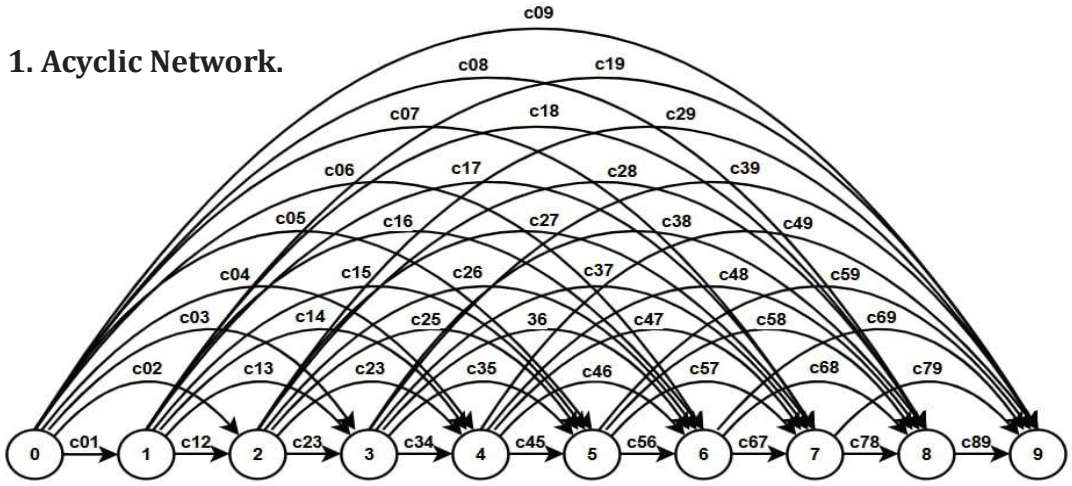
r_{j-i+1} — total repair costs during the period of use;

v_{j-i+1} — residual value obtained at the end of the service period.

To represent all decision alternatives, a special directed graph is constructed (Figure 1), called an acyclic network. The nodes represent years, and the arcs indicate the service periods of new equipment. Each arc is labeled with the corresponding cost value.

In the figure, the years of the planning horizon are represented as nodes. Each arc indicates that the equipment is purchased in a given year and used until subsequent years. The value on the arc represents the cost incurred during this period in the form of $c_{i-1\,j}$. The shortest path from the initial node to the final node provides the optimal strategy for equipment replacement and maintenance.

Figure 1. Acyclic Network.



Dynamic Programming Recursion

We consider the remaining planning horizon starting from year n. Let f_n denote the minimal total cost for the period from year n to year **N**. Then, the recursive equation is written as follows:

$$f_n = \min_{\substack{j=n,\dots N \\ P_i}} [c_{n-1\,j} + f_{j+1}],\ f_{N+1} = 0 \qquad (2)$$

In this formula, represents the minimum cost for all subsequent decisions after year n, and is obtained from the table constructed above. Using this formula, the optimal sequence of decisions is calculated for each year, and as a result, the most efficient strategy is determined for the 9-year plan—indicating when to replace the dental compressors and when to continue operating them through maintenance.

PROBLEM. Modern dental clinics use expensive equipment, and one of the most frequently used devices is the dental compressor. The operational planning horizon for a dental compressor is 9 years; at the beginning of each year, it is possible either to replace the equipment with a new unit or to continue using the existing one. At the beginning of year i, the purchase price of a new unit is sums; after j years of use, the worn equipment can be sold for v_j sums. To operate the new equipment consecutively for j years requires repair costs of r_j sums. All prices given in the problem are in US dollars.

Using the data provided in the table below:

1.Compute the costs c_{ij} and construct the corresponding table;

2.Construct the corresponding acyclic network (DAG);

3.Determine the optimal replacement strategy for the new equipment that yields the minimal total cost;

4.Calculate the optimal minimal cost value.

P	115	120	125	130	135	140	150	155	165
v	90	85	70	60	55	45	40	20	0
r	20	25	40	55	100	140	170	240	280

Solution. First, we calculate the costs $c_{i-1\,j}$ using formula $c_{i-1\,j} = P_i + r_{j-i+1} - v_{j-i+1}$:

ISSN: 3060-4761

When $i=1, j=1$,
$c_{01} = P_1 + r_1 - v_1 = 115 + 20 - 90 = 45$

When $i=1, j=2$,
$c_{02} = P_1 + r_2 - v_2 = 115 + 25 - 85 = 55$

When $i=1, j=3$,
$c_{03} = P_1 + r_3 - v_3 = 115 + 40 - 70 = 85$

When $i=1, j=4$,
$c_{04} = P_1 + r_4 - v_4 = 115 + 55 - 60 = 110$

When $i=1, j=5$,
$c_{05} = P_1 + r_5 - v_5 = 115 + 100 - 55 = 160$

When $i=1, j=6$,
$c_{06} = P_1 + r_6 - v_6 = 115 + 140 - 45 = 210$

When $i=1, j=7$,
$c_{07} = P_1 + r_7 - v_7 = 115 + 170 - 40 = 245$

When $i=1, j=8$,
$c_{08} = P_1 + r_8 - v_8 = 115 + 240 - 20 = 335$

When $i=1, j=9$,
$c_{09} = P_1 + r_9 - v_9 = 115 + 280 - 0 = 395$

When $i=2, j=2$,
$c_{12} = P_2 + r_1 - v_1 = 120 + 20 - 90 = 50$

When $i=2, j=3$,
$c_{13} = P_2 + r_2 - v_2 = 120 + 25 - 85 = 60$

When $i=2, j=4$,
$c_{14} = P_2 + r_3 - v_3 = 120 + 40 - 70 = 90$

When $i=2, j=5$,
$c_{15} = P_2 + r_4 - v_4 = 120 + 55 - 60 = 115$

When $i=2, j=6$,
$c_{16} = P_2 + r_5 - v_5 = 120 + 100 - 55 = 165$

When $i=2, j=7$,
$c_{17} = P_2 + r_6 - v_6 = 120 + 140 - 45 = 215$

When $i=2, j=8$,
$c_{18} = P_2 + r_7 - v_7 = 120 + 170 - 40 = 250$

When $i=2, j=9$,
$c_{19} = P_2 + r_8 - v_8 = 120 + 240 - 20 = 340$

When $i=3, j=3$,
$c_{23} = P_3 + r_1 - v_1 = 125 + 20 - 90 = 55$

When $i=3, j=4$,
$c_{24} = P_3 + r_2 - v_2 = 125 + 25 - 85 = 65$

When $i=3, j=5$,
$c_{25} = P_3 + r_3 - v_3 = 125 + 40 - 70 = 95$

When $i=3, j=6$,
$c_{26} = P_3 + r_4 - v_4 = 125 + 55 - 60 = 120$

When $i=3, j=7$,
$c_{27} = P_3 + r_5 - v_5 = 125 + 100 - 55 = 170$

When $i=3, j=8$,
$c_{28} = P_3 + r_6 - v_6 = 125 + 140 - 45 = 220$

When $i=3, j=9$,
$c_{29} = P_3 + r_7 - v_7 = 125 + 170 - 40 = 255$

When $i=4, j=4$,
$c_{34} = P_4 + r_1 - v_1 = 130 + 20 - 90 = 60$

When $i=4, j=5$,
$c_{35} = P_4 + r_2 - v_2 = 130 + 25 - 85 = 70$

When $i=4, j=6$,
$c_{36} = P_4 + r_3 - v_3 = 130 + 40 - 70 = 100$

When $i=4, j=7$,
$c_{37} = P_4 + r_4 - v_4 = 130 + 55 - 60 = 125$

When $i=4, j=8$,
$c_{38} = P_4 + r_5 - v_5 = 130 + 100 - 55 = 175$

When $i=4, j=9$,
$c_{39} = P_4 + r_6 - v_6 = 130 + 140 - 45 = 225$

When $i=5, j=5$,
$c_{45} = P_5 + r_1 - v_1 = 135 + 20 - 90 = 65$

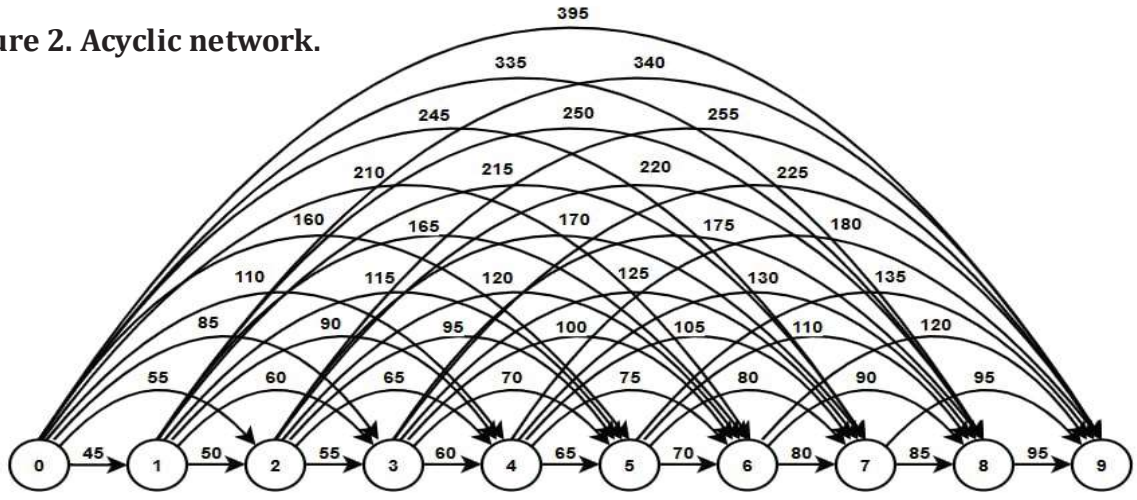
When $i=5, j=6$,
$c_{46} = P_5 + r_2 - v_2 = 135 + 25 - 85 = 75$

When $i=5, j=7$,
$c_{47} = P_5 + r_3 - v_3 = 135 + 40 - 70 = 105$
When $i=5, j=8$,
$c_{48} = P_5 + r_4 - v_4 = 135 + 55 - 60 = 130$
When $i=5, j=9$
$c_{49} = P_5 + r_5 - v_5 = 135 + 100 - 55 = 180$
When $i=6, j=6$,
$c_{56} = P_6 + r_1 - v_1 = 140 + 20 - 90 = 70$
When $i=6, j=7$,
$c_{57} = P_6 + r_2 - v_2 = 140 + 25 - 85 = 80$
When $i=6, j=8$,
$c_{58} = P_6 + r_3 - v_3 = 140 + 40 - 70 = 110$
When $i=6, j=9$,
$c_{59} = P_6 + r_4 - v_4 = 140 + 55 - 60 = 135$
When $i=7, j=7$,
$c_{67} = P_7 + r_1 - v_1 = 150 + 20 - 90 = 80$
When $i=7, j=8$,
$c_{68} = P_7 + r_2 - v_2 = 150 + 25 - 85 = 90$
When $i=7, j=9$,
$c_{69} = P_7 + r_3 - v_3 = 150 + 40 - 70 = 120$
When $i=8, j=8$,
$c_{78} = P_8 + r_1 - v_1 = 155 + 20 - 90 = 85$
When $i=8, j=9$,
$c_{79} = P_8 + r_2 - v_2 = 155 + 25 - 85 = 95$
When $i=9, j=9$,
$c_{89} = P_9 + r_1 - v_1 = 165 + 20 - 85 = 100$

We place the resulting $c_{i-1\,j}$ values into the acyclic network (Figure 2).

Now we express the $c_{i-1\,j}$ costs in tabular form (Table 1). This table, in turn, helps us determine the values of f_n. If we place the results obtained above into the table, we get:

Figure 2. Acyclic network.



Now, based on the table above and using the $f_n = \min_{j=n,\ldots N}[c_{n-1\,j} + f_{j+1}]$, $f_{N+1} = 0$ formula, we determine the optimal minimal cost: $f_{N+1} = f_{10} = 0$

$$f_9 = \min_{j=9}[c_{89} + f_{10}] = \min_{j=9}[95 + 0] = 95$$

ISSN: 3060-4761
TerDPI ILMIY XABARNOMASI / SCIENTIFIC BULLETIN OF THE TerSPI / НАУЧНЫЙ ВЕСТНИК ТерГПИ

	1	2	3	4	5	6	7	88 8	9 9
0	45	55	85	110	160	210	245	335	395
1		50	60	90	115	165	215	250	340
2			55	65	95	120	170	220	255
3				60	70	100	125	175	225
4					65	75	105	130	180
5						70	80	110	135
6							80	90	120
7								85	95
8									95

Table 1. Cost table for the period from year i to year j.

Minimal path: $8 \Rightarrow 9$

$$f_8 = \min_{j=8,9}[c_{78} + f_9; c_{79} + f_{10};] = \min_{j=8,9}[85+95; 95+0] = \min_{j=8,9}[180; 95] = 95$$

Minimal path: $7 \Rightarrow 9$

$$f_7 = \min_{j=7,8,9}[c_{67} + f_8; c_{68} + f_9; c_{79} + f_1\ ] = \min_{j=7,8,9}[80+95; 90+95; 120+0] = \min_{j=7,8,9}[175; 185; 120] = 120$$

Minimal path: $6 \Rightarrow 9$

$$f_6 = \min_{j=6,7,8,9}[c_{56} + f_7; c_{57} + f_8; c_{58} + f_9; c_{59} + f\ ] = \min_{j=6,7,8,9}[70+120; 80+95; 110+95; 135+0] = 135$$

Minimal path: $5 \Rightarrow 9$

$$f_5 = \min_{j=5,6,7,8,9}[c_{45} + f_6; c_{46} + f_7; c_{47} + f_8; c_{48} + f_9; c_{49} + f\ ] = \min_{j=5,6,7,8,9}[65+135; 75+120; 105+95; 130+95; 180+0] = 180$$

Minimal path: $4 \Rightarrow 9$

$$f_4 = \min_{j=4,5,6,7,8,9}[c_{34} + f_5; c_{35} + f_7; c_{36} + f_7; c_{37} + f_8; c_{38} + f_9; c_{39} + f_{10}] = \min_{j=4,5,6,7,8,9}[60+180; 70+135;$$

$$100+120; 125+95; 175+95; 225+0] = 205$$

Minimal path: $3 \Rightarrow 5$

$$f_3 = \min_{j=34,5,6,7,8,9}[c_{23} + f_4; c_{24} + f_5; c_{25} + f_7; c_{26} + f_7; c_{27} + f_8; c_{28} + f_9; c_{29} + f_{10}] = \min_{j=3,4,5,6,7,8,9}[55+205; 65+180;$$

$$95+135; 120+120; 170+95; 220+95; 255+0] = 230$$

Minimal path: $2 \Rightarrow 5$

$$f_2 = \min_{j=2,3,4,5,6,7,8,9}[c_{12} + f_3; c_{13} + f_4; c_{14} + f_5; c_{15} + f_7; c_{16} + f_7; c_{17} + f_8; c_{18} + f_9; c_{19} + f_1\ ] =$$

$$= \min_{j=2,3,4,5,6,7,8,9}[50+230; 60+205; 90+180; 115+135; 165+120; 215+95; 250+95; 340+0] = 250$$

Minimal path: $1 \Rightarrow 5$

$$f_1 = \min_{j=1,2,3,4,5,6,7,8,9}[c_{01} + f_2; c_{02} + f_3; c_{03} + f_4; c_{04} + f_5; c_{05} + f_7; c_{06} + f_7; c_{07} + f_8; c_{08} + f_9; c_{09} + f_{10}] =$$

$$= \min_{j=1,2,3,4,5,6,7,8,9}[45+250; 55+230; 85+205; 110+180; 160+135; 210+120; 245+95; 335+95; 395+0] = 285$$

Minimal path: $0 \Rightarrow 2$

From the results above, it is evident that the optimal minimum-cost path is $0 \Rightarrow 2 \Rightarrow 5 \Rightarrow 9$. If we verify the optimal minimum-cost path using the cost table, option $0 \Rightarrow 2 \Rightarrow 5 \Rightarrow 9 = 55 + 95 + 135 = 285$ records the result, and since in this example our minimal value matches the value obtained above, this confirms that the example was solved correctly. Now let us verify our example using Microsoft Excel (Figure 3) As a result of the calculations, the optimal strategy for equipment replacement and maintenance was identified. Using the dynamic programming method, the minimum total cost and the corresponding replacement years were determined. The results were verified using Excel, and they matched, indicating the correctness of the model and its practical applicability.

	A	B	C	D	E	F	G	H	I	J	K	L
1	№	1	2	3	4	5	6	7	8	9		
2	P	115	120	125	130	135	140	150	155	165		
3	v	90	85	70	60	55	45	40	20	0		
4	r	20	25	40	55	100	140	170	240	280		
5												
6		i-yildan j-yilgacha bo'lgan davr uchun harajatlar jadvali										
7	№	1	2	3	4	5	6	7	8	9		
8	0	45	55	85	110	160	210	245	335	395		
9	1		50	60	90	115	165	215	250	340		
10	2			55	65	95	120	170	220	255		
11	3				60	70	100	125	175	225		
12	4					65	75	105	130	180		
13	5						70	80	110	135		
14	6							80	90	120		
15	7								85	95		
16	8									95		
17												
18		n-yildan oxirigacha optimal minimal xarajat									Minimal qiymatlar	Yonalish
19	№	1	2	3	4	5	6	7	8	9		
20	10										0	
21	9									95	95	8=>9
22	8								180	95	95	7=>9
23	7							175	185	120	120	6=>9
24	6						190	175	205	135	135	5=>9
25	5					200	195	200	225	180	180	4=>9
26	4				240	205	220	220	270	225	205	3=>5
27	3			260	245	230	240	265	315	255	230	2=>5
28	2		280	265	270	250	285	310	345	340	250	1=>5
29	1	295	285	290	290	295	330	340	430	395	285	0=>2
30												
31	Optimal yo'li:		0=>2=>5=>9			Optimal yechim:		285				

Figure 3. Solution in Excel.

CONCLUSION. In conclusion, the results of the study show that the mathematical model developed for minimizing replacement and maintenance costs provides effective outcomes for the 9-year planning period. Based on the model, it was determined when it is optimal to replace the equipment and when it is more reasonable to continue maintenance. The calculations demonstrated that in some years the purchase of new equipment and in other years the repair of existing equipment can significantly reduce total costs. The analysis performed in Excel made it possible to identify the optimal replacement strategy, and the results confirmed the practical relevance of the model.

Furthermore, a brief sensitivity analysis of the parameters showed that if maintenance costs increase rapidly, earlier replacement becomes necessary, whereas higher residual values make it economically preferable to use the equipment for a longer period. This serves as an important practical indicator in real decision-making processes within production enterprises. Overall, the developed mathematical optimization model is an effective scientific and practical tool for managing long-term assets, improving production efficiency, and reducing costs in enterprises.

REFERENCES:

1.Ahmedov, U. Sh., & Abdukarimov, B. X. (2019). Operatsiyalar tadqiqi asoslari. Toshkent: O'qituvchi.

2.Taha, H. A. (2017). Operations Research: An Introduction (10th ed.). Pearson Education.

3.Nazarov, B. B. (2018). Iqtisodiy modellashtirish asoslari. Toshkent: Fan va texnologiya.

4.Hamidov, U. M., & Yunusov, A. A. (2016). Sanoat korxonalarida texnologik rejalashtirish. Toshkent: Ma'rifat.

5.Hillier, F. S., & Lieberman, G. J. (2010). Introduction to Operations Research (9th ed.). McGraw-Hill Education.

6.Khojiev, B. X. (2015). Korxonani boshqarish va iqtisodiy tahlil. Toshkent: Iqtisodiyot.

7.Bahromov, K. K. (2020). Iqtisodiy modellar va ularning kompyuterda yechilishi. Toshkent: Innovatsiya.

8.Winston, W. L. (2004). Operations Research: Applications and Algorithms (4th ed.). Cengage Learning.

9.Mamatova, Z. X. (2021). Texnologik tizimlarda ekspluatatsiya xarajatlarini minimallashtirish masalalari. Toshkent: Fan.